



K.R. MANGALAM UNIVERSITY

THE COMPLETE WORLD OF EDUCATION

Roll no.....

End Semester Examination, December 2024

B.Sc. (H) Mathematics – 3rd Semester

Real Analysis (SCMA201A)

Time: 2.5 hrs.

Max Marks: 50

Note: *All Questions are compulsory. Marks are indicated against each question.*

PART-A

Q.1 *First question will consist of 7 small questions covering entire syllabus out of which student has to attempt 5 questions and understanding level will be assessed.*

- (a) Define the supremum and infimum of a nonempty subset of $\mathbb{R}$. [CO-1] [L-1] 2
- (b) State the Archimedean property of the real numbers [CO-1] [L-1] 2
- (c) Define an open set and a closed set in $\mathbb{R}$. [CO-1] [L-1] 2
- (d) State the Bolzano-Weierstrass theorem for sequences. [CO-1] [L-1] 2
- (e) State the Fundamental Theorem of Integral Calculus. [CO-1] [L-1] 2
- (f) Explain the difference between pointwise and uniform convergence of a sequence of functions. [CO-1] [L-1] 2
- (g) Define an improper integral and give an example. [CO-1] [L-1] 2

2x5

PART-B

There will be five questions each covering one specific Course Outcome from the complete syllabus. There shall be an internal choice in each of the questions covering the same (set) of COs and BL. Questions carrying 10 marks can be further sub divided as per the paper setter choice.

Q.2 If the series $\sum u_n$ converges, then $\lim_{n \rightarrow \infty} u_n = 0$. And Show that the series $\sum_{n=1}^{\infty} \frac{n}{n+1}$ is not convergent.

[CO-2] [L-2] 10

OR

- (i) Prove that an arbitrary union of open sets is open. [CO-2] [L-3] 10
- (ii) Prove that every bounded sequence has a limit point. Converse may or may not be true. [CO-2] [L-3] 10

Q.3 Prove that if a function f is monotonic on $[a, b]$, then it is Riemann integrable on $[a, b]$.

Show that $3x+1$ is integrable on $[1, 2]$ and $\int_1^2 (3x+1) dx = \frac{11}{2}$. [CO-4] [L-3] 10

OR

- i. Prove that every continuous function is Riemann integrable. [CO-4] [L-4] 10
- ii. If a function f is bounded and integrable on $[a, b]$, then the function F defined a

$F(x) = \int_a^x f(t) dt, (a \leq x \leq b)$ is continuous on $[a, b]$. Further if f is continuous in $[a, b]$, then $F' = f$.

[CO-4] [L-4] 10

Q.4 Explain the convergence and non-convergence of the sequence. And find the behaviour of a_n

where, $a_n = 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{1}{2^{n-1}}$ and find $\lim_{n \rightarrow \infty} a_n$.

[CO-3] [L-5] **10**

OR

State and prove cauchy's general principle of uniform convergence. Show that the sequence

$\{S_n\}_{n=1}^{\infty}$, where $S_n = \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+n}$ is convergent.

[CO- 3]

[L-4] **10**

Q.5 Test the convergence of the following series:

(i) $\sum_{n=1}^{\infty} \left(\frac{2^2}{1^2} - \frac{2}{1} \right) + \left(\frac{3^3}{2^3} - \frac{3}{2} \right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3} \right)^{-3} + \dots$

[CO-5] [L-6]

10

OR

Test the convergence of the following series:

i. $\frac{x}{1.3} + \frac{x^2}{2.4} + \frac{x^3}{3.5} + \dots$

ii $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n\sqrt{n}}$.

[CO- 5] [L-6] **10**