



Roll no.....

End Semester Examination, December 2024
B.Sc. (Hons/Hons with research)
Mathematics – I Semester
Classical Algebra (SCMA103)

Time: 2.5 hrs.

Max Marks: **50**

Note: **All Questions are compulsory.** Marks are indicated against each question.

PART-A

Q.1 Attempt any five Questions out of Seven.

- (a) Write the relation between cube roots of unity. [CO-1] [L-2] **2**
- (b) Differentiate between remainder theorem and factor theorem. [CO-1] [L-4] **2**
- (c) Define bijective mapping with example. [CO-1] [L-1] **2**
- (d) Justify with illustration that congruence relation is an equivalence relation. [CO-1] [L-4] **2**
- (e) Give example of linear dependent and linear independent set of vectors. [CO-1] [L-3] **2**
- (f) State Cayley Hamilton Theorem. [CO-1] [L-1] **2**
- (g) Apply elementary row operation $3R_2$ on the Matrix $A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 4 & 5 & 3 & 4 \\ 1 & 4 & 5 & 6 \end{bmatrix}$ [CO-1] [L-3] **2**

2x5

PART-B

- Q.2 Find the Characteristics values and Characteristics vectors of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$ [CO-4] [L-3] **10**

OR

- i) Reduce the given matrix A to the row-echelon form and hence find the rank of A.

$$A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix} \quad [\text{CO-4}][\text{L-4}] \quad \mathbf{05}$$

- ii) Verify Cayley Hamilton Theorem for the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ [CO-4][L-4] **05**

Q.3 Solve the following equations

- (i) $x^5 + x^4 + x^3 + x^2 + x + 1 = 0$
- (ii) $x^6 - x^5 + x^4 - x^3 + x^2 - x + 1 = 0$ [CO-1] [L-2] **10**

OR

- Q.4 i) Differentiate between Division algorithm and Euclidean algorithm with examples. [CO-3] [L-2] **05**
 ii) Find GCD (12378, 3054) [CO-3] [L-2] **05**

OR

- i) Justify Fundamental theorem of Arithmetic with at least two examples. [CO- 3] [L-4] **05**
 ii) State and prove all properties of congruence relation. [CO- 3] [L-4] **05**

- Q.5 Let $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Let R_1 be a relation on X given by $R_1 = \{(x, y) : x - y \text{ is divisible by } 3\}$ and R_2 be a relation on X given by
 $R_2 = \{(x, y) : \{x, y\} \subset \{1, 4, 7\} \vee \{x, y\} \subset \{2, 5, 8\} \vee \{x, y\} \subset \{3, 6, 9\}\}$. Show that $R_1 = R_2$.
 [CO-1] [L-1] **10**

OR

- i) Show that the function $f: R \rightarrow R$ defined by $f(x) = 3x^3 + 5$ for all $x \in R$ is a bijection. [CO-2] [L-3] **5**
 ii) Find $f \circ g \wedge g \circ f$, if $f: R \rightarrow R \wedge g: R \rightarrow R$ are given by $f(x) = |x| \wedge g(x) = |5x - 3|$.
 [CO-2][L-3] **5**