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End Semester Examination, December 2024
B.Sc(H) Mathematics – V Semester
Set Theory and Metric Spaces(BSMA303A)

Time: 2.5 hrs.

Max Marks: **50**

Note: **All Questions are compulsory.** Marks are indicated against each question.

PART-A

Q.1 *Attempt any Five Questions.*

- (a) Give one example of each of the following:-
i) One-One Function
ii) on to function [CO-1] [L-2]
- (b) Fill in the blanks with appropriate words/symbols/signs:
(i) $(A \cap B)^{\circ} = \underline{\hspace{2cm}}$.
(ii) $d(x, y), d(x, z) + d(z, y)$ [CO-1] [L-1] **2**
- (c) List all properties of semi-metric space [CO-1] [L-1] **2**
- (d) Define diameter of a subset and distance between point and Subset. [CO-1] [L-1] **2**
- (e) Write the metric function of $R^p \vee l_p$ space. [CO-1] [L-2] **2**
- (f) Identify one example of countable and uncountable set each. [CO-1] [L-2] **2**
- (g) Differentiate between interior point and exterior point. [CO-1] [L-4] **2**

2x5

PART-B

- Q.2 If A and B are two denumerable sets, then prove that $A \times B$ is denumerable. [CO-2] [L-2] **10**

OR

Prove that union of two countable sets is Countable. [CO-2][L-2] **10**

- Q.3 Show that the following notations of Cantor Set are equivalent to each other:-
i) $C = \bigcap_{n=1}^{\infty} C_n$ [CO-3] [L-3] **10**
ii) $C = [0,1] - \bigcup_{n=1}^{\infty} E_n$

OR

Discover the open sphere $S((0,0),1)$ for the metric

- i) $d(z_1, z_2) = |x_1 - x_2| + |y_1 - y_2|$
ii) $d(z_1, z_2) = \sqrt{x_1^2 + y_1^2 + x_2^2 + y_2^2}$ [CO-3] [L-3] **10**
Where $z_1 = (x_1, y_1), z_2 = (x_2, y_2) \in R^2$

- Q.4 i) Illustrate with example that every convergent sequence in a metric space is a Cauchy sequence but converse need not be true. [CO-4] [L-4] **05**
ii) A Cauchy sequence is convergent iff it has a convergent subsequence. Justify the statement. [CO-4] [L-4] **05**

OR

Write a short note on the following:

- i) Covers
ii) Compact Metric spaces
iii) Bolzano Weierstrass Property
iv) Sequentially Compact Metric Spaces
v) Finite intersection property [CO- 4] [L-3] **2*5**
- Q.5 Prove that every compact metric space is complete. [CO-5] [L-2] **10**

OR

Prove that a continuous image of a connected space is connected.

[CO- 5] [L-2] **10**