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End Semester Examination, December 2024
Program – B.Tech, 1st Semester
ENGINEERING CALCULUS (ENMA101)

Time: 2.5 hrs.

Max Marks: **50**

Note: **All Questions are compulsory. Marks are indicated against each question.**

PART-A

Q.1 *Attempt any Five Questions.*

- (a) State Gauss divergence theorem. [CO-4] [L-1] **2**
- (b) Find the n th differential coefficient of $\frac{1}{ax+b} \left(x \neq -\frac{b}{a}\right)$. [CO-1] [L-1] **2**
- (c) Find the first order partial derivatives of following questions :
- (i) $u = y^x$ (ii) $u = \log(x^2 + y^2)$. [CO-2] [L-1] **2**
- (d) Explain homogenous function. Give one example. [CO-2] [L-2] **2**
- (e) Find a unit vector normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point (1, 2, -1). [CO-4] [L-1] **2**
- (f) Change the cartesian co-ordinates to cylindrical co-ordinates. Also explain the Jacobian. [CO-3] [L-2] **2**
- (g) Find : $\lim_{x \rightarrow \infty} \left(\frac{2x^3 - 3x^2 + 2}{x^3 - x^2 - 100x + 1} \right)$. [CO-1] [L-1] **2**

2x5

PART-B

Q.2 Examine the continuity and derivability of the function 'f' defined by
 $f(x) = \begin{cases} x^2 \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ [CO-1] [L-4] **10**

OR

Identify whether the following series is convergent or divergent?

- (a) $\sum_{n=1}^{\infty} \frac{e^{-n}}{n + \cos^2 n}$
- (b) $\sum_{n=1}^{\infty} \frac{n^n}{3^{1+2n}}$
- (c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ [CO-1][L-3] **10**

Q.3 (a) If $x = r \cos \theta$, $y = r \sin \theta$, find $J = \frac{\partial(x, y)}{\partial(r, \theta)}$ and $J' = \frac{\partial(r, \theta)}{\partial(x, y)}$. Show that $J J' = 1$. [CO-2] [L-3] **10**

(b) If $u = \cos \left(\frac{xy + yz + zx}{x^2 + y^2 + z^2} \right)$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$. [CO-2] [L-3] **10**

OR

Examine the extreme values $f(x, y) = x^2 - xy + y^2 + 3x - 2y + 1$. [CO-2] [L-4] **10**

- Q.4 Examine the smaller of the areas bounded by the ellipse $4x^2 + 9y^2 = 36$ and the straight line $2x + 3y = 6$. [CO-3] [L-4] **10**

OR

- (a) Change the order of integration in $\int_0^a \int_y^a \frac{xdxdy}{x^2 + y^2}$ and hence solve the same. [CO-3] [L-3] **10**
- (b) Solve $\int_0^1 \int_0^1 \frac{dxdy}{\sqrt{(1-x^2)(1-y^2)}}$. [CO-3] [L-3] **10**

- Q.5 (a) State Green's theorem and using the theorem to solve $\oint_C (x^2 y dx + x^2 dy)$, where C is the boundary described counterclockwise of the triangle with vertices $(0, 0)$, $(1, 0)$ and $(1, 1)$. [CO-4] [L-1,3] **10**

- (b) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. [CO-4] [L-1] **10**

OR

- (a) Examine the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point $P(1, 2, 3)$ in the direction of the line PQ where Q is the point $(5, 0, 4)$. In what direction it will be maximum? Find also the magnitude of this maximum. [CO-4] [L-4] **10**
- (b) Identify the divergence and curl of the vector $\vec{f} = e^{xyz}(x y^2 \mathbf{i} + y z^2 \mathbf{j} + z x^2 \mathbf{k})$ at the point $(1, 2, 3)$. [CO-4] [L-3] **10**